1. Uspořádej vzestupně podle velikosti čísla: $2^2, (-2)^3, -(-2)^2, -(-2)^3, -(-2^4)$

$2^2 = 4$

$(-2)^3 = -8$

$-(-2)^2 = -4$

$-(-2)^3 = -(-8) = 8$

$-(-2^4) = -(-16) = 16$

$\Rightarrow 2^2, (-2)^3, -(-2)^2, -(-2)^3, -(-2^4)$

2. a) $(-2)^3 \cdot (-1)^3 \cdot (-4)^2 = (-8)\cdot(-1)\cdot 16 = 128$

b) $-(-3)^2 \cdot (-1)^3 \cdot (-2)^3 \cdot (-4) = (-9)\cdot(-1)\cdot(-8)\cdot(-4) = 288$

3. a) $2a^3 - 3(-a)^5 + (-a)^3 + 2(-a)^5 - a^5 = 2a^3 - 3(-a^5) - a^3 + 2(-a^5) - a^5 = 2a^3 + 3a^5 - a^3 - 2a^5 - a^5 = a^3$

b) $(-b)^7 + 2(-b)^8 - 3(-b)^7 + (-b)^9 - b^8 - 2b^7 + b^9 = -b^7 + 2b^8 - 3(-b^7) + (-b^9) - b^8 - 2b^7 + b^9 =$
$b^8 + 3b^7 - 3b^7 - b^9 + b^9 = b^8$

c) $(-x)^2(-x)^4 + x^9 : x^3 + 3(x^2)^3 - 5(-x)^6 = x^2 \cdot x^4 + x^6 + 3x^6 - 5x^6 = x^6 - x^6 = 0$

4. Vyjádři pomocí mocnin prvočísel

a) $\dfrac{(-2)^4(-2)^5}{(-2^2)^3 \cdot (-2^2)} = \dfrac{2^4 \cdot (-2^5)}{-(2^2)^3 \cdot (-2^2)} = \dfrac{-2^9}{-2^6 \cdot (-2^2)} = \dfrac{-2^9}{2^8} = -2$

b) $\dfrac{(2^4 \cdot 3^2)^3}{(2^3 \cdot 3)^5} \cdot \dfrac{(-2^2)^3}{8} = \dfrac{2^{12} \cdot 3^6}{2^{15} \cdot 3^5} \cdot \dfrac{-2^6}{2^3} = -\dfrac{2^{12} \cdot 3^6}{2^{15} \cdot 3^5} \cdot \dfrac{2^6}{2^3} = -3$

c) $\left(\dfrac{2^2}{5}\right)^5 \left(-\dfrac{5^2}{2^3}\right)^3 = \dfrac{2^{10}}{5^5} \cdot \left(-\dfrac{5^6}{2^9}\right) = -2 \cdot 5$

d) $\dfrac{(2^2 - |-5|)^{19}}{(-2)^3(-5)^2} = \dfrac{(4-5)^{19}}{-2^3 \cdot 5^2} = -\dfrac{(-1)^{19}}{2^3 \cdot 5^2} = -\dfrac{-1}{2^3 \cdot 5^2} = 2^{-3} \cdot 5^{-2}$

5. Spočti a urči podmínky, za kterých mají výrazy smysl:

a) $\dfrac{(x+y)^3(x+y)^4}{(x+y)^5} : (x+y)^2 = \dfrac{(x+y)^7}{(x+y)^5} : (x+y)^2 = 1$

$x \neq y$

b) $\dfrac{a^3b^3a^4b^5}{ab^4a^4b^2} \cdot \dfrac{1}{a^2b^2} = \dfrac{a^7b^8}{a^5b^6} \cdot \dfrac{1}{a^2b^2} = 1$

$a \neq 0; b \neq 0$

c) $\dfrac{3a^2b^3}{4c^5} \cdot \dfrac{2c^3b^2}{9a^4} \cdot \dfrac{6a^3c^3}{b^4} = \dfrac{3 \cdot 2 \cdot 6}{4 \cdot 9} \cdot \dfrac{a^5b^5c^6}{a^4b^4c^5} = abc$

$a \neq 0; b \neq 0; c \neq 0$

d) $\dfrac{x^2y^3z^4}{m^5} \cdot \dfrac{m^4z}{2x^3y^2} : \dfrac{z^4}{4m^2x} = \dfrac{yz^5}{2mx} \cdot \dfrac{4m^2x}{z^4} = 2myz$

$m \neq 0; x \neq 0; y \neq 0; z \neq 0;$

e) $\dfrac{(-v)^3(-v)^5}{v^2} \cdot \dfrac{v^5}{(-v)^2} = \dfrac{-v^3 \cdot (-v^5) \cdot v^5}{v^2 \cdot v^2} = v^9$

$v \neq 0$

f) $\frac{m^3(-m)^4}{m(-m)^5}:\frac{(-m)^5}{m^2(-m)^3}=\frac{m^3\cdot m^4}{m\cdot(-m^5)}:\frac{-m^5}{m^2\cdot(-m^3)}=\frac{m^7}{-m^6}:1=-m$

$m\neq 0$

g) $(x-y)^3(y-x)^4=(x-y)^3\left[-(x-y)\right]^4=(x-y)^3(x-y)^4=(x-y)^7$

$x\neq y$

h) $\frac{(r-s)^2(s-r)^2}{r^2s^2}\cdot\frac{r^6s^3}{r-s}=\frac{(r-s)^2(-[r-s])^2r^6s^3}{r^2s^2(r-s)}=(r-s)(r-s)^2r^4s=(r-s)^3r^4s$

$r\neq 0; s\neq 0; r\neq s;$

6. Spočti a urči podmínky, za kterých mají výrazy smysl

a) $\frac{a^6(a^3)^2}{(a^3)^4}=\frac{a^6\cdot a^6}{a^{12}}=1$

$a\neq 0$

b) $\left(\frac{-2c}{d}\right)^2\left(\frac{-2d}{c}\right)^3\left(\frac{c}{2}\right)^4=\frac{(-2)^2c^2}{d^2}\cdot\frac{(-2)^3d^3}{c^3}\cdot\frac{c^4}{2^4}=2^2\cdot\frac{-2^3d}{c}\cdot\frac{c^4}{2^4}=-2c^3d$

$c\neq 0; d\neq 0$

c) $\left[16a^4\left(\frac{1}{a^2}\right)^3\left(\frac{a}{2}\right)^4\right]^3=\left[2^4a^4\frac{1}{a^6}\frac{a^4}{2^4}\right]^3=\left[a^2\right]^3=a^6$

$a\neq 0$

d) $\left(\frac{a^2x^3}{a^3x^2}\right)^4\left(\frac{a}{x}\right)^5=\left(\frac{x}{a}\right)^4\left(\frac{a}{x}\right)^5=\frac{x^4a^5}{a^4x^5}=\frac{a}{x}$

$a\neq 0; x\neq 0;$

e) $\frac{\left[3(z^2)^3y^2\right]^2}{(3zy)^4}=\frac{\left[3z^6y^2\right]^2}{3^4z^4y^4}=\frac{3^2z^{12}y^4}{3^4z^4y^4}=\frac{z^8}{3^2}$

$z\neq 0; y\neq 0$

f) $\frac{2^5(2b^3x^3)^2}{2(2bx^2)^3}=\frac{2^52^2b^6x^6}{2\cdot 2^3b^3x^6}=2^3b^3=(2b)^3$

$b\neq 0; x\neq 0$

g) $\frac{(2p^2q^3)^4(3q^2r)^3(25r^2p^3)^2}{(6p^3r)^3(2\cdot 5pq^4)^4}=\frac{2^4p^8q^{12}\cdot 3^3q^6r^3\cdot 5^4r^4p^6}{2^3\cdot 3^3p^9r^3\cdot 2^4\cdot 5^4p^4q^{16}}=\frac{pq^2r^4}{2^3}=\frac{pq^2r^4}{8}$

$p\neq 0; q\neq 0; r\neq 0;$

$\frac{(2u^3v^2)^4}{12}:\frac{(2u^2v^5\cdot 3u^4v^2)^2}{(3u^2v^4)^3}=\frac{2^4u^{12}v^8}{2^2\cdot 3}:\frac{2^2u^4v^{10}\cdot 3^2u^8v^4}{3^3u^6v^{12}}=\frac{2^2u^{12}v^8}{3}:\frac{2^2u^6v^2}{3}=$

h) $\frac{2^2u^{12}v^8}{3}\cdot\frac{3}{2^2u^6v^2}=u^6v^6=(uv)^6$

$u\neq 0; v\neq 0$

7. Vyjádři pomocí mocnin prvočísel

$$\frac{125^2\cdot 2^{10}}{16^2\cdot 50}\cdot\frac{25}{5^3}=\frac{\left(5^3\right)^2\cdot 2^{10}}{\left(2^4\right)^2\cdot 2\cdot 5^2}\cdot\frac{5^2}{5^3}=\frac{5^6\cdot 2^{10}\cdot 5^2}{2^8\cdot 2\cdot 5^5}=2\cdot 5^3$$

a) $125=5\cdot 25=5^3$

$50=2\cdot 25=2\cdot 5^2$

$$\left(\frac{128\cdot 3^5}{81\cdot 8}\right)^3\cdot\frac{9^4}{\left(16\cdot 3^5\right)^2}=\left(\frac{2^7\cdot 3^5}{3^4\cdot 2^3}\right)^3\cdot\frac{\left(3^2\right)^4}{\left(2^4\cdot 3^5\right)^2}=\left(2^4\cdot 3\right)^3\cdot\frac{3^8}{2^8\cdot 3^{10}}=\frac{2^{12}\cdot 3^3\cdot 3^8}{2^8\cdot 3^{10}}=2^4\cdot 3$$

b) $128=4\cdot 32=2^2\cdot 4\cdot 8=2^7$

$81=3\cdot 27=3^4$

$$\frac{\left(72\cdot 25\right)^3\cdot 2^4}{5\cdot 15^4\cdot 32^2}\cdot\left(\frac{5^2\cdot 27^2\cdot 8^3}{\left[12\cdot 5\right]^3\cdot 9}\right)^2=\frac{\left(2^3\cdot 3^2\cdot 5^2\right)^3\cdot 2^4}{5\cdot\left(3\cdot 5\right)^4\cdot\left(2^5\right)^2}\cdot\left(\frac{5^2\cdot\left(3^3\right)^2\cdot\left(2^3\right)^3}{\left[3\cdot 2^2\cdot 5\right]^3\cdot 3^2}\right)^2=$$

c) $$\frac{2^9\cdot 3^6\cdot 5^6\cdot 2^4}{5\cdot 3^4\cdot 5^4\cdot 2^{10}}\cdot\left(\frac{5^2\cdot 3^6\cdot 2^9}{3^3\cdot 2^6\cdot 5^3\cdot 3^2}\right)^2=3^2\cdot 2^3\cdot 5\cdot\left(\frac{3\cdot 2^3}{5}\right)^2=3^2\cdot 2^3\cdot 5\cdot\frac{3^2\cdot 2^6}{5^2}=\frac{3^4\cdot 2^9}{5}$$

$72=8\cdot 9=2^3\cdot 3^2$

$32=2^5$

8. Zjednoduš výrazy a urči podmínky:

a) $$\frac{2\left(ab\right)^3}{3a^2b}\cdot\frac{\left(3a^3b^2\right)^2}{a^5b^3}=\frac{2a^3b^3}{3a^2b}\cdot\frac{3^2a^6b^4}{a^5b^3}=6a^2b^3$$

$a\neq 0; b\neq 0$

b) $$\frac{5a^3b^7}{2ab^6}\cdot\left(\frac{2a^2b^3}{ab^2}\right)^3=\frac{5a^3b^7}{2ab^6}\cdot\frac{2^3a^6b^9}{a^3b^6}=5\cdot 2^2a^5b^4$$

$a\neq 0; b\neq 0$

c) $$\frac{2x^5y^3}{\left(2x^2y\right)^2}:\left(\frac{xy}{2xy^2}\right)^3=\frac{2x^5y^3}{2^2x^4y^2}:\left(\frac{1}{2y}\right)^3=\frac{xy}{2}:\frac{1}{2^3y^3}=\frac{xy}{2}\cdot 2^3y^3=2^4xy^4$$

$x\neq 0; y\neq 0$

d) $$\frac{7x^4y^7}{8x^3y}:\frac{\left(x^2y\right)^4}{\left(2x^3y^2\right)^3}=\frac{7xy^6}{8}:\frac{x^8y^4}{2^3x^9y^6}=\frac{7xy^6}{2^3}:\frac{1}{2^3xy^2}=\frac{7xy^6}{2^3}\cdot 2^3xy^2=7x^2y^8$$

$x\neq 0; y\neq 0$

e) $$\frac{a-b}{\left(a+b\right)^{-1}}=\frac{a-b}{\frac{1}{a+b}}=\left(a-b\right)\left(a+b\right)=a^2-b^2$$

$a\neq -b$

f) $$\left(\frac{a^{-3}b^2}{c^{-3}d}\right)^{-2}=\frac{a^6b^{-4}}{c^6d^{-2}}=\frac{a^6d^2}{c^6b^4}$$

$a\neq 0; b\neq 0; c\neq 0; d\neq 0;$

g) $$\left(\frac{3x^{-2}y^{-3}}{5z^{-4}}\right)^{-5}=\frac{3^{-5}x^{10}y^{15}}{5^{-5}z^{20}}=\frac{5^5x^{10}y^{15}}{3^5z^{20}}=\frac{5^5}{3^5}\frac{x^{10}}{z^{20}}y^{15}=\left(\frac{5}{3}\right)^5\left(\frac{x}{z^2}\right)^{10}y^{15}$$

$x\neq 0; y\neq 0; z\neq 0;$

h) $\left(\frac{x^0 z^{-3}}{y^{-3}}\right)^{-4} = \left(\frac{z^{-3}}{y^{-3}}\right)^{-4} = \frac{z^{12}}{y^{12}} = \left(\frac{z}{y}\right)^{12}$

$x \neq 0; y \neq 0; z \neq 0;$

$\left(\frac{a^2 b^{-4}}{c^{-3} d^{-2}}\right)^{-3} : \left(\frac{a^3 b^{-3}}{c^{-2} d^{-2}}\right)^{-2} = \frac{a^{-6} b^{12}}{c^9 d^6} \cdot \left(\frac{a^3 b^{-3}}{c^{-2} d^{-2}}\right)^2 = \frac{b^{12}}{a^6 c^9 d^6} \cdot \left(\frac{a^3 b^{-3}}{c^{-2} d^{-2}}\right)^2 =$

i) $\frac{b^{12}}{a^6 c^9 d^6} \cdot \frac{a^6 b^{-6}}{c^{-4} d^{-4}} = \frac{b^{12}}{a^6 c^9 d^6} \cdot \frac{a^6 c^4 d^4}{b^6} = \frac{b^6}{c^5 d^2}$

$a \neq 0; b \neq 0; c \neq 0; d \neq 0;$

j) $\left[\frac{1}{(x+y)^{-3}}\right]^{-2} \cdot (x+y)^{-3} = \frac{1}{(x+y)^6} \cdot (x+y)^{-3} = \frac{1}{(x+y)^9}$

$x \neq -y$

$\left(\frac{a^{-3} b^{-7} c^0}{a^{-5} b^{-11} c^{13}}\right)^{-4} \left(\frac{a^2 b^{-3} c^{-4}}{a^4 b^7 c^0}\right)^{-2} = \left(a^{-3-(-5)} b^{-7-(-11)} c^{0-13}\right)^{-4} \left(a^{2-4} b^{-3-7} c^{-4-0}\right)^{-2} = \left(a^2 b^4 c^{-13}\right)^{-4} \left(a^{-2} b^{-10} c^{-4}\right)^{-2} =$

k) $a^{-8} b^{-16} c^{52} \cdot a^4 b^{20} c^8 = a^{-4} b^4 c^{60} = \frac{b^4 c^{60}}{a^4}$

$a \neq 0; b \neq 0; c \neq 0;$

$\left(a + \frac{1}{b}\right)^{-2} \left(b - \frac{1}{a}\right)^{-3} \left(ab - \frac{1}{ab}\right)^2 = \left(\frac{ab}{b} + \frac{1}{b}\right)^{-2} \left(\frac{ab}{a} - \frac{1}{a}\right)^{-3} \left(\frac{a^2 b^2}{ab} - \frac{1}{ab}\right)^2 =$

l) $\left(\frac{ab+1}{b}\right)^{-2} \left(\frac{ab-1}{a}\right)^{-3} \left(\frac{a^2 b^2 - 1}{ab}\right)^2 = \left(\frac{b}{ab+1}\right)^2 \left(\frac{a}{ab-1}\right)^3 \left(\frac{(ab+1)(ab-1)}{ab}\right)^2 =$

$\frac{b^2}{(ab+1)^2} \frac{a^3}{(ab-1)^3} \frac{(ab+1)^2 (ab-1)^2}{a^2 b^2} = \frac{a}{ab-1}$

$a \neq \pm \frac{1}{b}$

9. Vypočti:

a) $2^{-3} - 4^{-2} - 5^{-2} + 20^{-2} = \frac{1}{2^3} - \frac{1}{4^2} - \frac{1}{5^2} + \frac{1}{20^2} =$

$\frac{1}{8} - \frac{1}{16} - \frac{1}{25} + \frac{1}{400} = \frac{50 - 25 - 16 + 1}{400} = \frac{10}{400} = \frac{1}{40}$

b) $\left(\sqrt{5} - 2\right)^{-1} = \frac{1}{\sqrt{5} - 2} = \frac{1}{\sqrt{5} - 2} \cdot \frac{\sqrt{5} + 2}{\sqrt{5} + 2} = \frac{\sqrt{5} + 2}{5 - 4} = \sqrt{5} + 2$

c) $40\left(5 - \sqrt{5}\right)^{-2} = \frac{40}{\left(5 - \sqrt{5}\right)^2} = \frac{40}{25 - 2 \cdot 5\sqrt{5} + 5} = \frac{40}{30 - 10\sqrt{5}} = \frac{40}{10\left(3 - \sqrt{5}\right)} =$

$\frac{4}{\left(3 - \sqrt{5}\right)} = \frac{4}{\left(3 - \sqrt{5}\right)} \cdot \frac{3 + \sqrt{5}}{3 + \sqrt{5}} = \frac{4\left(3 + \sqrt{5}\right)}{9 - 5} = \frac{4\left(3 + \sqrt{5}\right)}{4} = 3 + \sqrt{5}$

d) $\left(\frac{1}{2}\right)^{-3}-\left(\frac{1}{4}\right)^{-2}-\left(\frac{1}{5}\right)^{-2}+\left(\frac{1}{20}\right)^{-2}=2^3-4^2-5^2+20^2=8-16-25+400=367$

e)
$$\left(\sqrt{3}\right)^{-2}-\left(-\sqrt{3}\right)^{-2}-\left(\frac{1}{\sqrt{3}}\right)^{3}-\left(-\frac{1}{\sqrt{3}}\right)^{3}=\frac{1}{\left(\sqrt{3}\right)^2}-\frac{1}{\left(-\sqrt{3}\right)^2}-\frac{1}{\left(\sqrt{3}\right)^3}-\frac{1}{-\left(\sqrt{3}\right)^3}=$$
$$\frac{1}{3}-\frac{1}{3}-\frac{1}{\left(\sqrt{3}\right)^3}+\frac{1}{\left(\sqrt{3}\right)^3}=0$$

10. Spočti (využij převedení čísla na exponenciální tvar)

a) $\frac{105000}{0,021}=\frac{105\cdot 10^3}{21\cdot 10^{-3}}=5\cdot 10^6$

b) $\frac{0,252}{70000}\cdot\frac{200}{0,9}=\frac{252\cdot 10^{-3}\cdot 2\cdot 10^2}{7\cdot 10^4\cdot 9\cdot 10^{-1}}=\frac{28\cdot 2\cdot 10^{-1}}{7\cdot 10^3}=8\cdot 10^{-4}$

c) $\frac{0,000575}{2300000}:\frac{0,005}{100000}=\frac{575\cdot 10^{-6}}{23\cdot 10^5}:\frac{5\cdot 10^{-3}}{10^5}=\frac{575\cdot 10^{-6}}{23\cdot 10^5}\cdot\frac{10^5}{5\cdot 10^{-3}}=\frac{115}{23}\cdot 10^{-3}=5\cdot 10^{-3}$

d) $800000\cdot 0,025=8\cdot 10^5\cdot 25\cdot 10^{-3}=200\cdot 10^2=2\cdot 10^4$

11. Zjednoduš výrazy (uveď podmínky)

a) $\frac{2^n\cdot 2^{n+1}\cdot 2^{n+2}}{2^3\cdot 2^2}=\frac{2^{n+n+1+n+2}}{2^5}=\frac{2^{3n+3}}{2^5}=2^{3n-2}$

b) $\frac{(-3)^{2n}\cdot(-3)^{2n+1}\cdot(-2)^{2n-1}}{-3}=\frac{3^{2n}\cdot -3^{2n+1}\cdot\left(-2^{2n-1}\right)}{-3}=-\frac{3^{4n+1}\cdot 2^{2n-1}}{3}=-3^{4n}\cdot 2^{2n-1}$

c) $\frac{a^n\cdot b^n\cdot a^n\cdot b^n}{ab}=\frac{a^n\cdot a^n\cdot b^n\cdot b^n}{ab}=\frac{a^{2n}\cdot b^{2n}}{ab}=\frac{(ab)^{2n}}{ab}=(ab)^{2n-1}=a^{2n-1}\cdot b^{2n-1}$

$a\neq 0; b\neq 0$

d) $(-a)^{2n+3}:a^{2n}=-a^{2n+3}:a^{2n}=-a^{2n+3-2n}=-a^3$

$a\neq 0$

e)
$$(a-b-c)^{2n}(b+c-a)^{2n+1}(b+c-a)=\left[-(b+c-a)\right]^{2n}(b+c-a)^{2n+2}=$$
$$(b+c-a)^{2n}(b+c-a)^{2n+2}=(b+c-a)^{4n+2}$$

f) $\left(\frac{a^2}{b^3}\right)^k\left(\frac{c^2}{d^3}\right)^k\left(\frac{b^2d^3}{ac^2}\right)^k=\frac{a^{2k}}{b^{3k}}\frac{c^{2k}}{d^{3k}}\frac{b^{2k}d^{3k}}{a^kc^{2k}}=\frac{a^k}{b^k}=\left(\frac{a}{b}\right)^k$

$a\neq 0; b\neq 0; c\neq 0; d\neq 0;$

g)
$$\left[\frac{(x-5)^2}{x-2}\right]^k\left[\frac{x^2-4}{x-5}\right]^k=\frac{(x-5)^{2k}}{(x-2)^k}\frac{\left(x^2-4\right)^k}{(x-5)^k}=(x-5)^k\frac{\left[(x-2)(x+2)\right]^k}{(x-2)^k}=$$
$$(x-5)^k\frac{(x-2)^k(x+2)^k}{(x-2)^k}=(x-5)^k(x+2)^k$$

$x\neq 2; x\neq 5$

h) $\frac{\left(1-\frac{a-b}{a}\right)^k}{\left(1+\frac{a-b}{b}\right)^k}=\frac{\left(\frac{a-(a-b)}{a}\right)^k}{\left(\frac{b+(a-b)}{b}\right)^k}=\frac{\left(\frac{b}{a}\right)^k}{\left(\frac{a}{b}\right)^k}=\frac{\frac{b^k}{a^k}}{\frac{a^k}{b^k}}=\frac{b^{2k}}{a^{2k}}=\left(\frac{b}{a}\right)^{2k}$

$$\frac{x^{2k}-2x^k+1}{x^3}:\frac{x^2}{\left(x^k-1\right)^3}=\frac{\left(x^k-1\right)^2}{x^3}\cdot\frac{\left(x^k-1\right)^3}{x^2}=\frac{\left(x^k-1\right)^5}{x^5}$$

i) $x \neq 0$

$x \neq 0$

$$\frac{\left(a-b\right)^{x+y+1}\left(a+b\right)^{x+y+1}}{\left(a^2-b^2\right)^{x+y}}=\frac{\left(a-b\right)^{x+y+1}\left(a+b\right)^{x+y+1}}{\left[\left(a-b\right)\left(a+b\right)\right]^{x+y}}=\frac{\left(a-b\right)^{x+y+1}\left(a+b\right)^{x+y+1}}{\left(a-b\right)^{x+y}\left(a+b\right)^{x+y}}=$$

j) $\left(a-b\right)\left(a+b\right)=a^2-b^2$

$a \neq \pm b$